Term Information

Effective Term	Spring 2018

General Information

Course Bulletin Listing/Subject Area	Mathematics
Fiscal Unit/Academic Org	Mathematics - D0671
College/Academic Group	Arts and Sciences
Level/Career	Undergraduate
Course Number/Catalog	4570
Course Title	Applied Algebraic Topology
Transcript Abbreviation	Applied Alg. Top.
Course Description	This course will serve as an introduction to algebraic topology, with a view toward persistent homology of point clouds for applications to data analysis. Homology of simplicial complexes over a field with a focus on building up intuition about homology moving to a specialized notion of persistent homology of persistence modules. Real-world applications to data analysis will be provided.
Semester Credit Hours/Units	Fixed: 3

Offering Information

Length Of Course	14 Week, 8 Week
Flexibly Scheduled Course	Never
Does any section of this course have a distance education component?	No
Grading Basis	Letter Grade
Repeatable	No
Course Components	Lecture
Grade Roster Component	Lecture
Credit Available by Exam	No
Admission Condition Course	No
Off Campus	Never
Campus of Offering	Columbus

Prerequisites and Exclusions

Prerequisites/Corequisites	A C- or better in Math 2568 and Math 3345.
Exclusions	

Cross-Listings

Cross-Listings

Subject/CIP Code

Subject/CIP Code	27.0101
Subsidy Level	Baccalaureate Course
Intended Rank	Junior, Senior

Requirement/Elective Designation

The course is an elective (for this or other units) or is a service course for other units

Course Details

Course goals or learning objectives/outcomes

- Understand metric topology
- Understand simplicial complexes and their homology
- Understand persistent holmology
- Understand structures on spaces of barcodes

Content Topic List

- Metric topology
- Homology of simplicial complexes
- Point clouds and associated spaces
- Persistent holmology
- Persistance diagrams
- Structures on the space of barcodes
- Applications

Attachments

- App_Alg_Top_Syllabus.pdf: Syllabus

  (Syllabus. Owner: Husen,William J)

Comments

Workflow Information

Status	User(s)	Date/Time	Step
Submitted	Husen,William J	04/07/2017 11:44 AM	Submitted for Approval
Approved	Husen,William J	04/07/2017 11:45 AM	Unit Approval
Approved	Haddad,Deborah Moore	04/07/2017 01:06 PM	College Approval
Pending Approval	Nolen,Dawn Vankeerbergen,Bernadette Chantal Hanlin,Deborah Kay Jenkins,Mary Ellen Bigler	04/07/2017 01:06 PM	ASCCAO Approval

APPLIED ALGEBRAIC TOPOLOGY

Course Description. The course will serve as an introduction to algebraic topology, with a view toward persistent homology of point clouds for applications to data analysis. In order to keep the material accessible to a wide audience, an emphasis will be placed on homology of simplicial complexes over a field. We will focus on building up intuition about what homology measures through concrete examples. We will then move on to the more specialized notion of persistent homology of persistence modules. Real-world applications to data analysis will be provided.

Credit Hours. 3

Intended Audience The course is designed for junior and senior undergraduate mathematics majors. No prior knowledge of topology or abstract algebra will be assumed. Students with familiarity in these subjects are welcome, as there is not a significant overlap with the standard courses. The course will also be appropriate for computer science and data analytics majors with a strong math background.

Prerequisites. A C- or better in Math 2568 and Math 3345 or equivalent.

Grading. Grades for the course will be determined by weekly homework assignments (60%), a midterm project (15%) and a final project (25%). Students will be given choices of topics for the midterm and final projects. These will range from guided investigations into deeper mathematics than what is covered in lecture to programming projects.

Text. The course will roughly follow the recent survey article [1] by Gunnar Carlsson. Background material which does not appear in the survey paper will be supplemented by additional course notes.

Academic Misconduct Statement. It is the responsibility of the Committee on Academic Misconduct to investigate or establish procedures for the investigation of all reported cases of student academic misconduct. The term "academic misconduct" includes all forms of student academic misconduct wherever committed; illustrated by, but not limited to, cases of plagiarism and dishonest practices in connection with examinations. Instructors shall report all instances of alleged academic misconduct to the committee (Faculty Rule 3335-5-487). For additional information, see the Code of Student Conduct http://studentlife.osu.edu/csc/.

Disability Statement. Students with disabilities (including mental health, chronic or temporary medical conditions) that have been certified by the Office of Student Life Disability Services will be appropriately accommodated and should inform the instructor as soon as possible of their needs. The Office of Student Life Disability Services is located in 098 Baker Hall, 113 W. 12th Avenue; telephone 614- 292-3307, slds@osu.edu; slds.osu.edu.

Tentative Schedule

Week 1: Review of Linear Algebra I

- Course overview and motivation
- Vector spaces and subspaces over $\mathbb{R}$ and F_2
- Examples
- Basis and dimension
- Linear transformations and matrix representations
- Homework: Basic properties of vector spaces

Week 2: Review of Linear Algebra II

- Kernel and cokernel of a linear transformation
- Quotient vector spaces
- Inner product spaces
- Normed spaces, leading to first examples of metrics
- Homework: Calculations with linear transformations, properties of norms

Week 3: Metric Topology I

- Definition of a metric space
- Examples of metric spaces
- Open and closed sets
- Continuous maps between metric spaces
- Homework: Working with basic properties of metric spaces

Week 4: Metric Topology II

- Basic topological properties of metric spaces: connectedness, compactness
- Equivalence relations
- Homeomorphism
- How to distinguish metric spaces? Light introduction to the ideas of π_0, π_1
- Example: clustering in finite metric spaces via π_0
- Homework: Basic topological properties

Week 5: Homology of Simplicial Complexes I

- Motivation: distinguishing metric spaces through linear algebra
- Return to linear algebra: free vector spaces generated by a finite set
- Homology of simplicial complexes: develop intuition by working simple examples in detail; start with calculations over F_2
- Homework: Working with free vector spaces and some basic homology calculations over F_2

Week 6: Homology of Simplicial Complexes II

- Chain complexes of vector spaces and boundary maps
- Abstract definition of homology of a chain complex of vector spaces
- Rigorous definition of homology of a simplicial complex
- Normal forms for matrix pairs as an algorithm for computing homology
- Light introduction to functoriality: inclusion maps induce maps on homology
- Homework: Calculating homology of simple examples of simplicial complexes and proving basic properties of homology

Week 7: Homology of Simplicial Complexes III

- Informal discussion of extending homology to general metric spaces
- Homotopy and homotopy equivalence in metric spaces
- Contractibility
- Contractible simplicial complexes have trivial homology (statement without formal proof)
- Homework: Working with homotopies

Week 8: Point Clouds and Associated Spaces

- Motivation: why study point clouds?
- Examples of point clouds arising from real-world data
- Point clouds as finite metric spaces
- Single-linkage clustering
- Persistent sets
- Vietoris-Rips complex
- Mid term project due

Week 9: Persistent Homology I

- Homology of the Vietoris-Rips complex of a point cloud
- Persistence vector spaces: definitions of persistence vector space, linear transformations, sub-persistence vector space
- Finitely-presented persistence vector spaces
- Basic properties of persistence vector spaces
- Homework: more basic properties of persistence vector spaces

Week 10: Persistent Homology II

- Classification theorem for finitely-presented persistence vector spaces
- Demonstration: Javaplex for topological data analysis
- Homework: filling in details of the proof of the classification theorem

Week 11: Persistence Diagrams

- Barcodes and persistence diagrams
- Persistent homology algorithm
- Computational examples
- Examples and applications of barcodes in the literature

Week 12: Structures on the Space of Barcodes I

- Define bottleneck distance on Barcode space
- Define Gromov-Hausdorff distance on the space of finite metric spaces
- Discuss the stability theorem relating the two distances (without proof)
- Homework: fill in details showing the bottleneck distance and Gromov-Hausdorff distance are metrics

Week 13: Structures on the Space of Barcodes II

- Define interleaving distance
- Work with a variety of simple examples to develop intuition about interleaving distance
- Sketch the proof of the isometry theorem relating interleaving distance to bottleneck distance
- Homework: fill in some details of the proof of the isometry theorem

Week 14: Applications

- The last week will be spent studying specific applications to real-world data. This can be catered to interests of the students.
- Final project

References

[1] Carlsson, G., 2014. Topological pattern recognition for point cloud data. Acta Numerica, 23, p.289.